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Research Article

Financial risk modelling with normal and Laplace distribution

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ABSTRACT:

The primary objective of this article is to provide a comprehensive yet accessible introduction to stable distributions within the context of financial modelling. Traditional financial models often rely on the normal (Gaussian) distribution to analyze asset returns; however, this approach is inadequate in capturing the significant variations and extreme events observed in real-world financial markets. Financial returns frequently exhibit heavy tails and greater kurtosis, making it essential to adopt more sophisticated models that better represent these fluctuations. One such alternative is the Laplace distribution, a class of probability distributions characterized by heavy tails. This distribution provides a more accurate depiction of substantial price swings and extreme events, which are crucial for risk assessment and portfolio management. Furthermore, the Laplace distribution allows for a broader range of dependence structures, making it a valuable tool for financial analysts. By incorporating stable distributions, financial models can enhance predictive accuracy and risk evaluation strategies.

Keywords: Tail Risk, Value at Risk, Laplace and normal distribution.

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1. Introduction

The primary aim of this research is to develop models utilising asymmetric Laplace distributions, which have proven to be effective in modelling foreign exchange rates, interest rates, and variations in stock prices (Jayakumar, 2007). It integrates the asymmetric Laplace distribution and time-varying dynamics into both exponential smoothing and GARCH formulations. Backtesting results demonstrate a moderate improvement in VaR forecasting accuracy (Gerlach, 2013).

The Laplace distribution is extensively employed in financial risk modelling owing to its capacity to effectively depict financial data characterised by heavy tails, resilience to extreme values, and asymmetry. The utilisation of this approach proves to be highly advantageous in the modelling of exceptional occurrences such as market crashes or significant price fluctuations, which are frequently observed in financial markets. The Laplace distribution exhibits reduced sensitivity to outliers, rendering it a desirable choice for the modelling of financial data that contains outliers.

Furthermore, this technique can be employed to simulate asymmetric data, such as the returns of assets and other financial variables that exhibit asymmetric patterns. The utilisation of the Laplace distribution extends to stress testing and risk management, whereby it facilitates the evaluation of tail risk and the estimation of Value at Risk (VaR) and Conditional Value at Risk (CVaR). The utilisation of the prior distribution in Bayesian analysis for parameter estimation, namely within the field of finance, is also observed. The utilisation of the Laplace distribution is prevalent in option pricing and derivatives modelling, particularly in scenarios where there is a departure from normality in the asset price returns. Additionally, it facilitates the development of more precise models for non-normal returns. The suitability of the Laplace distribution is contingent upon the particular attributes of the data and the objectives of the research.

2. Literature Review

Financial markets are intricate systems characterised by significant oscillations that pose difficulties for conventional risk assessment methods. Tail risk, which refers to infrequent but highly consequential occurrences in financial markets, is frequently overlooked in traditional approaches to risk assessment (Jorion, 2007). Incorporating non-Gaussian models into the Value at Risk (VaR) calculation has been acknowledged as a transformative theoretical framework that addresses the limitations of Gaussian-based approaches and improves the efficacy of risk management measures (Beaulieu, 2007). Incorporating non-Gaussian models into Value at Risk (VaR) calculations enhances the efficacy of risk management strategies by incorporating a broad spectrum of risks, enabling institutions to mitigate potential losses effectively (Del Brio, 2020). This initiative facilitates financial stability and enhances compliance with regulatory standards. The capacity to conduct accurate evaluations of tail risk enables investors to navigate volatile markets and construct robust portfolios in the presence of calamitous events (Kadan, 2014).

Numerous studies have developed methodologies for estimating extreme conditional quantiles, independent component analysis, and non-Gaussian financial risk management procedures. Using fuzzy portfolio Value at Risk (VaR) and Expected Shortfall (ES) models to resolve non-Gaussian distributions has been proposed as an extension of Yoshida's extended model (Moussa, 2014). In the context of financial market crises, the q-Gaussian probability density function demonstrates greater precision in estimating Value at Risk (VaR) than standard models. Numerous studies have examined non-extensive statistical mechanics models within the context of finance's optimal portfolio selection problem (Devi, 2019).

These efforts have centred on incorporating a Value-at-Risk constraint. Comparisons have been made between the student-t distribution, autoregressive conditional heteroskedastic (ARCH) models, and extreme value theory (EVT) for depicting returns distributions with enormous tails (Lechner, 2010). Previous studies' reliance on the assumption of normality has the potential to produce misleading results. In conclusion, incorporating non-Gaussian models into Value at Risk (VaR) calculations represents a paradigm transition within the field of risk management. This method addresses the limitations inherent in conventional Gaussian-focused techniques (Tyralis, 2022).

3. Data and Methodology

The present study uses the daily returns of S&P BSE SENSEX for the period 01-04-2000 to 31-12-2022. The daily closing price S&P BSE SENSEX values are obtained from bseindia.com and the returns are calculated on the basis of the following- If the closing level of Sensex on date and be the same for its previous business day, i.e., omitting intervening weekend or stock exchange holidays, then the one-day return on the market portfolio is calculated as:

$$r_t = ln\frac{l_t}{l_{t-1}} X\ 100$$

Where LN (*CLOSE*) is the natural logarithm of '*CLOSE*.'

Normal distribution Probability Density Function:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \qquad (1)$$

Inverse of PDF of Normal distribution

$$x = F^{-1}(p \mid \mu, \sigma) = \{x: F(x \mid \mu, \sigma) = p\}, (2)$$

Laplace Distribution Probability Density Function

$$f(x) = \frac{1}{\sqrt{2}\sigma} \exp\left[-\frac{\sqrt{2}|x-\mu|}{\sigma}\right] \quad (3)$$

Inverse PDF of Laplace distribution

$$F(x) = \frac{1}{2} e^{\frac{x-\mu}{b}}, x \leq \mu \qquad (4)$$

$$2F(x) = e^{\frac{x-\mu}{b}}$$

$$\ln(2F(x)) = \frac{x-\mu}{b}$$

$$x = \mu + \text{b}\ln(2F(x))$$

Modelling Value at Risk (VaR) with the Laplace distribution entails estimating the potential losses a financial portfolio may incur over a specified time horizon with a given level of confidence. When financial data exhibits non-normality and fat-tailed behaviour, this method is particularly useful for modelling VaR. To implement VaR, you must acquire historical data, calculate daily returns or price changes, and ensure that the data is stationary. Estimate parameters using the Laplace distribution, which is distinguished by its location () and scale (b). Calculate VaR at the desired levels of confidence (e.g., 95%, 99%).

The calculated VaR represents the utmost potential loss at the specified level of confidence over the selected time frame. Backtesting and model validation are required to assess the VaR model's accuracy and make necessary adjustments. Evaluate the impact of extraordinary events that may not be captured by VaR alone by conducting scenario analysis and stress testing.

Utilise VaR estimates to inform risk management decisions, portfolio optimisation, and capital allocation within a business. While the Laplace distribution can capture fat-tailed behaviour and provide more realistic VaR estimates than the normal distribution, it still simplifies the financial markets' underlying complexity.

4. Analysis and Discussion

Table 1. Descriptive statistics of BSE 30 Index

count	5721
mean	0.000424
std	0.014354
min	-0.14102
25%	-1%
50%	0%
75%	1%
max	0.1599

Source: Author own calculation

The analysis of descriptive statistics table 1 indicates that the log return features do not adhere to the characteristics of a normal distribution. Therefore, it is necessary to consider the application of non-normal distribution in order to capture tail risk.

Figure 1. Log return of BSE 30 Index

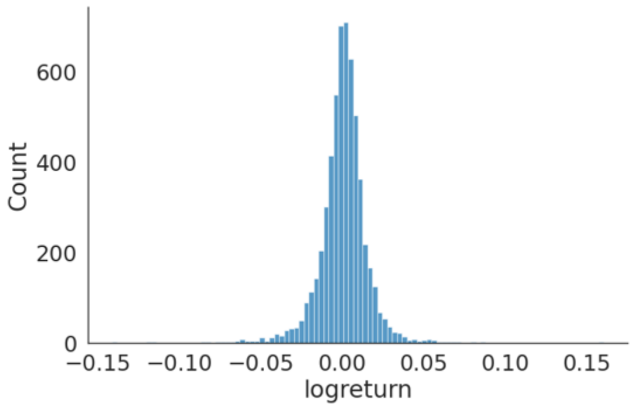

Table 2. The Distribution Fit information

Distributions	sumsquare_error	location	scale
Laplace	70.1974	0.00092	774.0682
Normal	724.928	0.000424	0.014352

Source: Author's own calculation

Error-Induced Square Sum, this statistic measures the total deviation between the response values and the model fit. It is also known as the square sum of residuals and is abbreviated as SSE. A closer value to 0 indicates a superior match. Table 2 indicates Laplace distribution is very close to the log return theoretical properties.

Figure 2. Distribution fit test

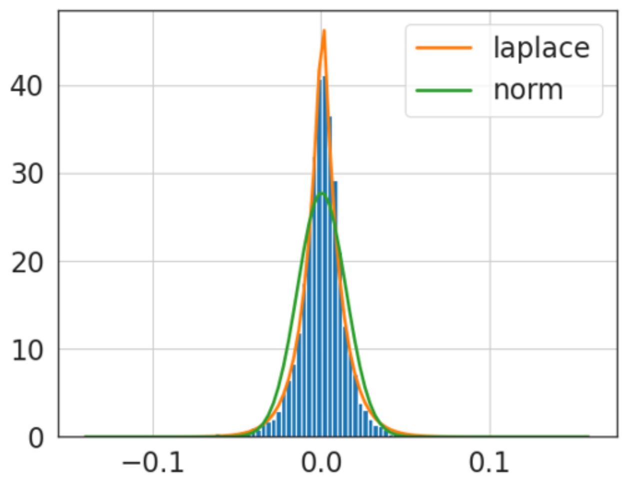

Figure 2 shows the distribution fit best at Laplace distribution and it covers more tail risk than normal distribution goodness of fit.

Table 3. The tail risk, if the investment of Rs. 100,000

			Normal distribution			
VaR	**1 Day**	**10 Days**	**30 Days**	**90 days**	**180 days**	**250 days**
0.25%	-119.056	-376.489194	-652.098	-1129.47	-1597.31	-1882.45
0.50%	-109.25	-345.479241	-598.388	-1036.44	-1465.74	-1727.4
1.00%	-98.6687	-312.017919	-540.431	-936.054	-1323.78	-1560.09
2.50%	-83.1291	-262.877229	-455.317	-788.632	-1115.29	-1314.39
5.00%	-69.7641	-220.613525	-382.114	-661.841	-935.984	-1103.07
10.00%	-54.3552	-171.886181	-297.716	-515.659	-729.251	-859.431
			Laplace distribution			
VaR	**1 Day**	**10 Days**	**30 Days**	**90 days**	**180 days**	**250 days**
0.25%	-487.445	-1541.43706	-2669.85	-4624.31	-6539.76	-7707.19
0.50%	-423.676	-1339.78007	-2320.57	-4019.34	-5684.21	-6698.9
1.00%	-359.906	-1138.12307	-1971.29	-3414.37	-4828.65	-5690.62
2.50%	-275.607	-871.547026	-1509.56	-2614.64	-3697.66	-4357.74
5.00%	-211.838	-669.890033	-1160.28	-2009.67	-2842.1	-3349.45
10.00%	-148.068	-468.233039	-811.003	-1404.7	-1986.54	-2341.17

Source: Author own calculation

Table 3 shows the tail risk (VaR) at various quantiles, with future risk prediction based on days one through two hundred and fifty. The results demonstrate conclusively that Laplace distribution provides more information about tail risk than normal distribution. The results also indicate that the use of various distributions improves the quality of risk reporting and provides investors and policymakers with more information about tail risk in the current market environment.

5. Conclusion

This article well explained stable distributions in financial models. The normal (or bell curve/Gaussian) model cannot represent asset variability, hence more precise models must be used to calculate financial returns. Dependent patterns and volatility are possible with heavy-tailed probability distributions like the Laplace distribution.

The Laplace distribution is commonly employed in modelling Value at Risk (VaR) within financial portfolios, enabling the estimation of probable losses within a predetermined period. This approach demonstrates utility in analyzing data that deviates from a normal distribution and exhibits fat-tailed characteristics. In order to operationalize Value at Risk (VaR), historical data is obtained, daily returns or price changes are computed, and parameters are evaluated utilizing the Laplace distribution. The computed Value at Risk (VaR) represents the upper bound of potential loss.

The Error-Induced Square Sum (SSE) quantifies the discrepancy between observed response values and the corresponding values predicted by a model. The Laplace distribution exhibits theoretical traits that closely resemble those of log returns, encompassing a greater degree of tail risk than the normal distribution. The findings indicate that the Laplace distribution offers greater insight into tail risk than the normal distribution. This enhancement in risk reporting quality equips investors and policymakers with a more comprehensive understanding of the associated risks.

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